

Intertial Frame Dragging in an Acoustic Analogue spacetime

Chandrachur Chakraborty*

Tata Institute of Fundamental Research, Mumbai 400005, India

Oindrila Ganguly†

S. N. Bose National Centre for Basic Sciences, Kolkata 700098, India

Parthasarathi Majumdar‡

Ramakrishna Mission Vivekananda University, Belur Math 711202, India.

We report an incipient exploration of the Lense-Thirring precession effect in a rotating *acoustic analogue black hole* spacetime. An exact formula is deduced for the precession frequency of a gyroscope due to inertial frame dragging, close to the ergosphere of a ‘Draining Bathtub’ acoustic spacetime which has been studied extensively for acoustic Hawking radiation of phonons and also for ‘superresonance’. Prospects of experimental detection of this new effect in the Draining bathtub acoustic geometry, are discussed.

INTRODUCTION

Measurements and observations of Lense-Thirring precession [1] of a gyroscope due to inertial frame dragging in a rotating black hole spacetime have been carried out only for spacetimes with curvatures small enough to justify the weak gravity approximation [2, 3]. It has not been possible so far to directly detect inertial frame dragging in regions with very strong gravitational fields close to rotating black holes and neutron stars [4, 5]. This is similar to other strong gravity phenomena like Hawking radiation or superradiance which are difficult to observe in realistic astrophysical situations, because of large background noise. In this respect, Unruh’s discovery [6] of Hawking radiation of phonons from acoustic black hole spacetimes, as well as the acoustic analogue of superradiance named as ‘superresonance’ [7] provide compelling acoustic analogue models of these intriguing phenomena which can be tested experimentally. A comprehensive review of the field of acoustic analogue spacetimes has been given by Visser [8]. This has now made it possible to design in the laboratory simple and robust models of various classes of Lorentzian geometry in kinematical situations (where the acoustic metric is non-dynamical).

In this letter, we explore a new physical effect in acoustic geometry - the Lense-Thirring precession due to frame dragging in a rotating acoustic black hole spacetime. Since the phenomenon is reasonably well-known in physical spacetime, and yet has never been observed except in its weak field version, acoustic analogue spacetimes afford us an arena for a more complete study where strong gravity aspects of this precession might be revealed in the laboratory. To this end, we shall restrict ourselves to a study of frame dragging close to the ergosphere of a particular fluid

mechanical analogue model of a rotating black hole, namely the so-called Draining Bathtub [8]. In terms of the fluid flow, the ergosphere characterises a transition surface from a sonic to a supersonic flow but sound waves from inside it may still travel outwards, away from the surface. The ergosphere enshrouds the acoustic horizon which as per standard practice is a one-way membrane for phonons. Adapting the derivation of the exact Lense-Thirring precession for four dimensional stationary spacetime discussed earlier in [4] (following the textbook [9]) to the three dimensional analogue spacetime being discussed here, an exact formula for the precession frequency appropriate to the Draining Bathtub black hole is given. The behaviour of the precession frequency as a function of the radial distance is briefly discussed and compared qualitatively with that found earlier for the Kerr black hole. It is also compared to the actual angular velocity of the background flow.

In regard to observational prospects and techniques, the difficulty of observing any precession effect in two space dimensions (as opposed to three) is well-known, since, in two dimensions, any angular momentum is a scalar quantity, while in three every precession effect is delineated by its own vector direction. Thus, in three dimensions, it is not hard to conceptualise a gyroscope which is free to precess with the rotating space. In two space dimensions, in contrast, all motions must be planar, including that of any gyroscope constructed out of the acoustic disturbance (phonons). This implies that the only possible effect of inertial frame dragging in this acoustic geometry of two spacelike dimensions, is a change of ‘orientation’ of the phonons in the background fluid. However, there is a potentially large background noise that can mask the manifestation of such a precessional motion of phonons, arising from their geodesic motion in the

acoustic geometry. We shall propose two possible scenarios for resolution of this conundrum, which may pave the way towards observation of this effect. This being the incipient paper on the Lense-Thirring effect (inertial frame dragging) in strong acoustic gravity, we make no claims of a full understanding about the observational details of the proposed effect.

ROTATING ACOUSTIC BLACK HOLE

The acoustic analogue of a rotating black hole spacetime is best captured by a planar ‘Draining Bathtub’ flow of an incompressible, barotropic, inviscid fluid with no global vortex present. The flow is characterised by the velocity potential

$$\vec{v}_b = -\frac{A}{r} \hat{r} + \frac{B}{r} \hat{\phi}. \quad (1)$$

Here, (r, ϕ) are plane polar coordinates while A, B are constants. The constraints imposed on the fluid make the background density ρ_b position independent which in turn guarantees the constancy of background pressure p_b and local speed of sound c_s throughout the fluid. As a simplifying measure, we set $c_s = 1$ and ignore an overall constant factor of $\frac{\rho_b}{c_s}$ in the acoustic metric. The explicit form of the emerging acoustic black hole metric is,

$$ds_{DB}^2 = -\left(1 - \frac{A^2 + B^2}{r^2}\right) dt^2 + dr^2 + r^2 d\phi^2 + \frac{2A}{r} dr dt - 2B d\phi dt. \quad (2)$$

It is clear that this curved analogue spacetime possesses isometries that correspond to time translations and rotations on the plane and hence, is not only stationary but also axisymmetric. The radius of the ergosphere, r_E , is determined by the vanishing of g_{00} : $r_E^2 = A^2 + B^2$. The 2-surface at $r_H = A$ acts as the future event horizon of the sonic black hole because just beyond it, the radial component of \vec{v}_b exceeds the local speed of sound. Any linearised fluctuation originating in the region bounded by the acoustic horizon is swept inward by the flow.

FRAME DRAGGING IN ‘DRAINING BATHTUB’ GEOMETRY

A general formula for the Lense-Thirring precession observed from a Copernican frame in four dimensional spacetime has been discussed in detail in [4]. This formula must now be adapted for an axially symmetric

(acoustic) three dimensional spacetime: one obtains the following expression for $\Omega_{(2+1)}$

$$\Omega_{(2+1)} = \frac{1}{2\sqrt{-g}} \epsilon_{ij} g_{00} \left[\frac{g_{0i}}{g_{00}} \right]_{,j} \quad (3)$$

To obtain the Lense-Thirring precession rate in Draining Bathtub case, we can easily apply Eq.(3) where i, j take values 1, 2 denoting the two spatial dimensions. For the line element Eq.(2) of an acoustic black hole,

$$\Omega_{(2+1)}^{DB} = -\frac{B r_E^2}{r^4} \left[1 - \frac{r_E^2}{r^2} \right]^{-1}. \quad (4)$$

It is clear from Eq. (4) that the the Lense-Thirring precession is more pronounced closer to the ergosphere $r \rightarrow r_E$ than away from it. This is precisely the sort of qualitative behaviour of gyroscopic precession observed in earlier work [4] on physical stationary axisymmetric spacetimes.

In the ‘weak field’, i.e., far away ($r \gg r_E$) from the ‘ergosphere’, $|\Omega_{(2+1)}^{DB}|$ decreases as $1/r^4$:

$$|\Omega_{(2+1)}^{DB}| \simeq \frac{B r_E^2}{r^4}, \quad \text{as } r \gg r_E. \quad (5)$$

This is also similar to the weak-field approximation that has been used for observation of the gyroscopic precession due to inertial frame dragging in the Earth’s gravitational field arising out of its diurnal rotation [10].

We now compare the Lense-Thirring precession angular frequency $\Omega_{(2+1)}^{DB}$ for the Draining Bathtub flow in $(2+1)$ dimensional acoustic spacetime, with the angular velocity of the flow itself, Ω , as a function of the radial distance. Observe that the angular velocity of the flow is given by,

$$\Omega(r) = \frac{B}{r^2} \quad (6)$$

On the other hand, the angular velocity of precession occurring due to dragging of local inertial frames has the form,

$$|\Omega_{(2+1)}^{DB}(r)| = \left[\Omega(r) \frac{r_E^2}{r^2} \left(1 - \frac{r_E^2}{r^2} \right)^{-1} \right], \quad (7)$$

after substituting the value of $\Omega(r) = \frac{B}{r^2}$ from Eq.(6). For convenience, we introduce a dimensionless radial coordinate $x \equiv r/r_E$ such that the ergosphere at $r = r_E$ maps to $x = 1$ while $r \rightarrow \infty$ corresponds to $x \rightarrow \infty$. Let us also define $\bar{\Omega} \equiv |\Omega_{(2+1)}^{DB}|/\Omega$ which evidently is dimensionless also. Equation (7) then has a simpler appearance:

$$\bar{\Omega}(x) = \frac{1}{x^2 - 1}$$

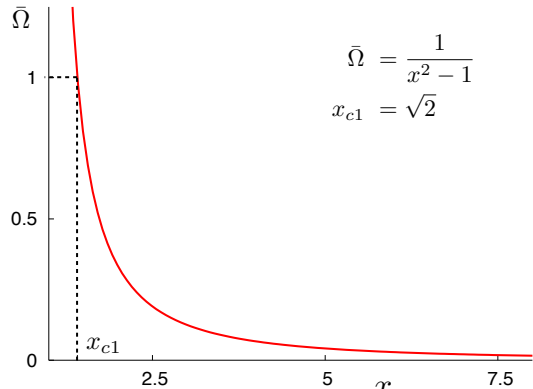


FIG. 1. Variation of $\bar{\Omega}$ with x showing the point x_{c1}

It is apparent that when $x = \sqrt{2}$, $\bar{\Omega} = 1$. Thus, at this point, the angular velocity of precession becomes equal in magnitude to the angular velocity of rotation before surpassing it with approach towards the ergosphere ($x \rightarrow 1$, $r \rightarrow r_E$). We denote the corresponding critical radius $r = \sqrt{2} r_E$ by r_{c1} and $x_{c1} = r_{c1}/r_E = \sqrt{2}$. This is graphically represented in FIG. 1.

OBSERVATIONAL PROSPECTS

Since all general relativistic effects in acoustic analogue gravity have to do with the behaviour of phonons in a curved acoustic spacetime, any prospect of observing inertial frame dragging in a Draining Bathtub Flow must focus on a change of ‘orientation’ or direction of motion of phonons in the two dimensional acoustic space. In this manner, the geodesic motion of phonons and their precessional motion due to inertial frame dragging are likely to get entangled, so that an ideal gyroscope that is free to precess relative to the Copernican frame is not immediately available. It also means that, as already pointed out, geodesic change of orientation plays the role of a large background noise for any attempt to discern the presence of inertial frame dragging.

However, note that, circular geodesic orbits exterior to the ergosphere have Keplerian frequencies that have a very different behaviour as a function of the radial distance *close* to the ergosphere, in comparison to the precession frequency due to inertial frame dragging. In particular, the Lense-Thirring precession frequency rises sharply from about twice the radius of the ergosphere, for ingoing phonons, and even exceeds the angular velocity of the flow at about $1.4r_E$. This is seen from Fig 1 as well as from the exact formula (4). In contrast, the Keplerian frequency for circu-

lar geodesic orbits exterior to the ergosphere of the Draining Bathtub flow is given by the formula

$$\Omega_K^{(DB)} = \frac{r_E}{r^2}. \quad (8)$$

As one approaches the ergosphere, the Keplerian frequency does not show any sharp enhancement around $r \rightarrow r_E+$. This remarkable distinction between the behaviour of circular geodesic orbits and the effect of the Lense-Thirring precession close to the ergosphere might provide a cue as to how the inertial frame dragging effect can become observable, *in principle*, very near the ergosphere. As one approaches the ergosphere, one should notice a significant *departure* of circular geodesic orbits away from circularity. According to our analysis, this departure ought to be inward, analogous to inspiralling of matter in an accretion disc around a compact object. The ‘signal’ due to Lense-Thirring precession, owing to its sharp enhancement close to the ergosphere, supercedes the background noise due to geodesic motion which shows no such enhancement.

An alternative scenario, albeit a bit speculative, may be conceptualised as follows : consider a super-radiant phonon emerging from the ergosphere. If its properties can be measured accurately, then it is possible to estimate the properties of an *ingoing* phonon which is identical to this superradiant phonon, except that it is ingoing. One can therefore kinematically prepare the ingoing phonon such that it collides head-on with the outgoing phonon in the absence of frame dragging, e.g., if the collision is arranged to take place a large radial distance away from the ergosphere. As a result of the collision, both phonons would deflect symmetrically about the point of collision. This collision phenomenon is now to be repeated closer and closer to the ergosphere, and the predicted effect is a gradual breakdown of the symmetry of deflection – the motion of two otherwise identical phonons, one ingoing and the other outgoing, will result in a deflection of the phonons with a bias towards the direction of the inertial frame dragging. Another equivalent manner of saying the same thing is: closer and closer to the ergosphere, the kinematical parameters of the ingoing phonon will have to be adjusted anew, if the collision with the outgoing phonon is to take place. Admittedly, the details of this particular scenario need to be worked out, but that will need another whole communication.

With Bose-Einstein condensate systems (BEC) providing the best experimental situations for more comprehensive studies of superfluidity and other aspects of low temperature physics, it is conceivable that they will also provide the arena for possible observation of inertial frame dragging in an acoustic ge-

ometry sensed by phonons in a rotating fluid. Starting with the time-dependent Gross-Pitaevski equation [11] which follows from the microscopic Bogoliubov equations for the order parameter for a BEC system exhibiting off-diagonal long range order in the mean-field approximation [12], the identification of the gradient of the phase of the order parameter with the velocity of the fluid leads to the Euler hydrodynamic equations [13–15]. The recent identification of an acoustic horizon [16] in such flows makes the prospects for the observation of the Lense-Thirring precession effect better than ever before.

We may make a final observation that the effect discussed in this paper is not directly related to the ‘Superfluid Gyroscope’ discerned by Stringari [17] where rotation of vortices in cold atomic gases is studied using this model. The frame dragging effect proposed by us for acoustic black holes is a property of the acoustic spacetime geometry as perceived by acoustic perturbations. It is very much within the paradigm of acoustic analogues of curved spacetime, and may or may not lead to any novel aspect of superfluids per se, as simulated by cold atom systems.

CONCLUSION

Kinemematical effects of phonon excitations of certain specific fluid flows hold great observational prospects in acoustic (or optical) analogues of ‘fixed metric’ gravitational phenomena. The possibility of experimental observation of inertial frame dragging for the first time, without having to invoke the ‘weak field approximation’ as one had to do for physical spacetime, is as exciting in our opinion as the observation of Hawking radiation of phonons first conceived of by Unruh in 1981. No doubt there are many details of the theory of cold atoms which have to be investigated to eventually actualise the conceptual elements discussed in this incipient essay, in a particular cold atom system, within its hydrodynamic domain. But since the physics of cold atoms in the hydrodynamic domain is fairly well understood, this is not an unsalable difficulty. Rather, it is not unlikely that what appears as inertial frame dragging from an analogue

gravity perspective, is a novel aspect of cold atom physics, or at any rate a novel perspective on a known phenomenon.

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* chandrachur.chakraborty@tifr.res.in

† lagubhai@gmail.com

‡ parthasarathi.majumdar@rkmvu.ac.in

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